NUMBER THEORY
PRACTICE EXERCISE

1. Find the remainder when 12231^{4321} is divided by 729
(1) 0 (2) 1
(3) 5 (4) 4
(5) None of these

2. Which of the following is definitely true?
(1) 0.01001000100001... is irrational
(2) $\frac{1}{3^2.7^3.11^7}$ and $\frac{1}{13^2.15^3 17^2}$ are both pure recurring
(3) Product of two irrational numbers is irrational
(4) Product of one rational and one irrational is rational
(5) None of these

3. Find the power of 153 in 1000!.
(1) 60 (2) 63
(3) 62 (4) 64
(5) None of these

4. Find the number of zeroes at the end of $[13!+14!]^{(13!+14!)}$
(1) $2^{(13!+14!)}$ (2) 2 (13! +14!)
(3) $3^{(13!+14!)}$ (4) 3 (13! +14!)
(5) 4 (13! +14!)

5. Find the remainder when $2^{37} + 6$ is divided by 74.
(1) 4 (2) 6
(3) 8 (4) 2
(5) 0

6. If x and y are two positive numbers x and y such that their sum is minimum and product is 30. Now if $N^2 = abxy$, where a and b are single digit numbers and $abxy$ represents a four digit number in decimal representation, find how many values can N take?
(1) 2 (2) 4
(3) 3 (4) 5
(5) 1

7. Find the remainder when 5^{2000} is divided by 1000.
(1) 25 (2) 125
(3) 625 (4) 75
(5) None of these

8. In how many ways 2100 can be represented as product of two numbers which are not co–prime to each other.
(1) 8 (2) 4
(3) 10 (4) 6
(5) 12

9. Find remainder when $(2222)^{5555} + (5555)^{2222}$ is divided by 7.
(1) 6 (2) 0
(3) 5 (4) 3
(5) None of these

10. Find the remainder when 3^{40} is divided by 23.
(1) 2 (2) 4
(3) 6 (4) 5
(5) 3

11. If p, $p + 4$ and $p + 14$ all three numbers are prime, find how many different possible values of 'p' are there?
(1) one (2) two
(3) three (4) Many
(5) None of these

12. If a, $a + 2$ and $a + 4$ all three numbers are prime, find how many different possible values of 'a' are there?
(1) one (2) two
(3) three (4) Many
(5) None of these

13. Find the remainder when $12^{83} + 14^{83}$ is divided by 169.
(1) 130 (2) 1
(3) 12 (4) 7
(5) 0

14. Find the remainder when $32^{32^{32}}$ is divided by 7.
(1) 4 (2) 2
(3) 0 (4) 6
(5) None of these

15. If m and n are integers divisible by 5, which of the following is not necessarily true ?
(1) $m - n$ is divisible by 5
(2) $m^2 - n^2$ is divisible by 25
(3) $m + n$ is divisible by 10
(4) $m + n$ is divisible by 5
(5) None of these

16. Let x, y and z be distinct integers. x and y are odd positive, and z is even and positive. Which one of the following statements cannot be true?
(1) $(x-z)^2 y$ is even (2) $(x-z)y^2$ is odd
(3) $(x-z)y$ is odd (4) $(x-y)^2 z$ is even
(5) None of these

17. The remainder obtained when $43^{101} + 23^{101}$ is divided by 66 is :
(1) 2 (2) 10
(3) 5 (4) 0
(5) 3

18. The last three–digits of the multiplication 12345×54321 will be
(1) 865 (2) 745
(3) 845 (4) 945

(5) 735

19. Today I bought a total of 735 chocolates and packed them into smaller packets, each packet containing an equal number of chocolate. I then distributed one packet each to all boys and one packet each to all the girl in a school. The boys got a total of 150 chocolates and the girls got a total of 240 chocolates. If tomorrow, I want to distribute the remaining packets, one each to all the students of this class, then find the minimum number of packets that I will fall short of?
(1) 7 (2) 6
(3) 5 (4) 3
(5) 4

20. Simplify $\cfrac{1}{3-\cfrac{1}{2-\cfrac{1}{1-\frac{2}{7}}}}$
(1) 1 (2) 0.75
(3) 0.5 (4) 0.25
(5) None of these

21. Which of the following statements is definitely false?
(1) (1111……..21 times) is a composite number
(2) (1111……..91 times) is a prime number
(3) Product of three consecutive odd integers is composite
(4) 111111…..198 times is divisible by 199.
(5) None of these

22. Find the remainder when $2 \times 26!$ is divided by 29.
(1) 1 (2) 27
(3) 3 (4) 26
(5) 28

23. Find the remainder when 19^{150} is divided by 95.
(1) 19 (2) 76
(3) 57 (4) 38
(5) 14

24. Find the smallest positive integer greater than 1, for which square root, cube root, fourth root and fifth root all are integer.
(1) 512 (2) 1024
(3) $(1024)^6$ (4) $(1024)^4$
(5) None of these

25. Find the least number which when divided by 52 leaves a remainder 19 but when divided by 51 leaves a remainder 11.
(1) 2295 (2) 2255
(3) 2355 (4) 2257
(5) None of these

26. If $f(n)$ = sum of the digits of n, Find the value of $f(100) + f(101) + \ldots f(199)$.
(1) 1000 (2) 900
(3) 1100 (4) 800
(5) None of these

27. If n is natural number, the number $n^4 + 2n^3 - n^2 - 2n$ is always multiple of:
(1) 60 (2) 120
(3) 24 (4) 18
(5) None of these

28. If HCF of $n^2 + 11$ and $(n + 1)^2 + 11$ is M, find the maximum value of M.
(1) 5
(2) 20
(3) 45
(4) Depends on the value of n
(5) None of these

29. Find the remainder when $1^7 + 2^7 + 3^7 + \ldots 100^7$ is divided by 33.
(1) 1 (2) 3
(3) 32 (4) 0
(5) None of these

30. Find the sum of perimeters of all rectangles whose sides are integral and area is 2000 cm^2.
(1) 4826 (2) 9652
(3) 2418 (4) 5000
(5) None of these

31. Find the number of factors of the number 23100, which are multiple of 5.
(1) 48 (2) 96
(3) 24 (4) 64
(5) None of these

32. If $f(n)$ represents number of zeroes at the end of $n!$, find number of possible values of n if $f(n) = 26$.
(1) 5 (2) 3
(3) 0 (4) 4
(5) None of these

33. Find the least number which when divided by 12,16,18,30 leaves remainder 4 in each case but it is completely divisible by 7?
(1) 2994 (2) 2864
(3) 3004 (4) 2784
(5) 2884

34. Find the remainder when 666……..2002 times is divided by 2002.
(1) 663 (2) 660
(3) 0 (4) 66
(5) None of these

35. Which of the following statements is/are always true?
(1) $n^2 + n + 41$ is always prime for all natural values of n.
(2) $2^m + 1$ is always prime for all m, where $m = 2^n$, where n is whole number.
(3) No of prime numbers are finite

(4) 111....57 times is prime
(5) None of these.

36. There are 30 apples, 45 bananas, 135 mangoes and 240 oranges. Find the minimum number of baskets required to pack these fruits such that each basket has equal number of fruits and no basket has fruits of more than one type.
(1) 30 (2) 15
(3) 20 (4) 40
(5) None of these

37. If x and y are integers, find the number of solutions of $xy = x + y + 359$
(1) 24 (2) 48
(3) 12 (4) 96
(5) 6

38. If a student always uses base system 5 for representation and computation of numbers, if he says that the number (1241320a32) is multiple of 4, find the value of a.
(1) 3 (2) 2
(3) 0 (4) 1
(5) 4

39. Find the simplified value of the expression $(7 + 1)(7^2 + 1)(7^4 + 1)(7^8 + 1)\ldots.(7^{1024} + 1)$
(1) $(7^{2096} - 1)/6$ (2) $(7^{2048} - 1)/6$
(3) $(7^{4096} - 1)/6$ (4) 7^{2049}
(5) None of these

40. Find the unit digit of the product $112^{123}.32^{11}.23^{564}.32^{554}.684^{5123}$
(1) 2 (2) 4
(3) 6 (4) 8
(5) None of these

41. Find the remainder when $1^{17} + 2^{17} + 3^{17} + \ldots 100^{17}$ is divided by 9.
(1) 8 (2) 3
(3) 2 (4) 0
(5) 1

42. In how many ways 100! can be represented as product of two numbers which are co–prime to each other.
(1) 2^{12} (2) $2^{24} - 1$
(3) 2^{24} (4) $2^{12} - 1$
(5) $2^{24} - 4$

43. Find the remainder when 1212121212.....100 digits is divided by 99.
(1) 60 (2) 66
(3) 0 (4) 6
(5) 4

44. Find the remainder when 123123123.....300 digits is divided by 999.
(1) 663 (2) 660
(3) 30 (4) 66
(5) 312

45. A three–digit positive integer abc is such that $a^2 + b^2 + c^2 = 74$. a is equal to the doubled sum of the digits in the tens and units places. Find the number if it is known that the difference between that number and the number written by the same digits in the reverse order is 495.
(1) 813 (2) 349
(3) 613 (4) 245
(5) None of these

46. If $x_1x_2x_3 = 1.25$ and $x_1x_2^2 = 5$, find the values of (x_1, x_2, x_3) so that $x_1 + x_2 + x_3$ is the minimum possible.
(1) $x_1 = 2.25, x_2 = 5, x_3 = 0.2$
(2) $x_1 = 1.25, x_2 = 4, x_3 = 4.5$
(3) $x_1 = 1.25, x_2 = 2, x_3 = 0.5$
(4) None of these
(5) Data insufficient

47. Let a, b, c, d be four integers such that $a + b + c + d = 4m + 1$ where m is a positive integer. Given m, which one of the following is necessary true?
(1) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - m + 1$
(2) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 + m + 1$
(3) The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - 2m + 1$
(4) The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 + 2m + 1$
(5) None of these

48. Find HCF of the numbers m and n where m, n are natural numbers and $113m - 12n = 1$
(1) 11 (2) Depends on m and n
(3) 1 (4) 3
(5) None of these

49. If a, b and c are natural numbers, find the minimum value of $2a + 3b + c$, where $a\ b\ c = 36$
(1) 36 (2) 18
(3) 12 (4) 10
(5) None of these

50. Find the highest 3 digit number which when divides 4000 and 6369 leaves same remainders.
(1) 100 (2) 123
(3) 103 (4) 23
(5) 133

ANSWER KEY

1.	(1)	**6.**	(3)	**11.**	(1)	**16.**	(1)	**21.**	(2)	**26.**	(1)	**31.**	(1)	**36.**	(1)	**41.**	(5)	**46.**	(3)
2.	(1)	**7.**	(3)	**12.**	(1)	**17.**	(4)	**22.**	(5)	**27.**	(3)	**32.**	(1)	**37.**	(2)	**42.**	(3)	**47.**	(2)
3.	(5)	**8.**	(3)	**13.**	(1)	**18.**	(2)	**23.**	(2)	**28.**	(3)	**33.**	(5)	**38.**	(2)	**43.**	(4)	**48.**	(3)
4.	(4)	**9.**	(2)	**14.**	(1)	**19.**	(4)	**24.**	(3)	**29.**	(1)	**34.**	(2)	**39.**	(2)	**44.**	(5)	**49.**	(2)
5.	(3)	**10.**	(1)	**15.**	(3)	**20.**	(2)	**25.**	(2)	**30.**	(5)	**35.**	(5)	**40.**	(2)	**45.**	(1)	**50.**	(3)